Statistics

Fall 2022

Lecture 17

Feb 19-8:47 AM

Binomial Prob. dist. SG 16

1) n independent events

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

3) p & q remain unchanged for all events

$p + q = 1$ $q = 1 - p$

4) x is # of successes, $p = \frac{x}{n}$, $x = np$

if decimal

⇒ Always Round-up

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Nov 21-6:00 AM

Consider a binomial Prob. dist with $n=12$, and $P=.6$.

1) $q = 1 - P = 1 - .6 = \boxed{.4}$

2) $np = 12(.6) = \boxed{7.2}$ 3) $npq = 12(.6)(.4) = \boxed{2.88}$

4) $\sqrt{npq} = \sqrt{2.88} \approx \boxed{1.697}$

5) $P(x=8) = {}_{12}C_{8} \cdot (.6)^{8} \cdot (.4)^{12-8} = \boxed{.213}$

$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

Using TI

2nd VARS ↓↓...↓ binompdf(n 12, P .6, x 8)

Trials: 12

P: .6

x-value: 8

Paste Enter

$= \boxed{.213}$

Nov 21-6:05 AM

Suppose we flip a fair coin 20 times and success is to land tails.

$n = 20$ $P = .5$ $q = 1 - P = \boxed{.5}$

$np = 20(.5) = \boxed{10}$ $npq = 20(.5)(.5) = \boxed{5}$

$\sqrt{npq} = \sqrt{5} \approx \boxed{2.236}$

$P(x=12) = {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^{20-12} = \boxed{.120}$

$P(x) = {}_{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

Now using TI

2nd VARS binompdf(n 20, P .5, x 12) =

$\boxed{P(x=12) = \text{binompdf}(20, .5, 12) = .120}$

$P(x \le 12) = P(x=12) + P(x=11) + P(x=10) + \cdots + P(x=0)$

$= \text{binomcdf}(20, .5, 12) = \boxed{.868}$

Nov 21-6:14 AM

Suppose you are taking a multiple-choice exam with 40 questions and each question has 4 choices with only one correct choice and you are making random guesses.

$n=40$ $\quad p=\frac{1}{4}=.25$ $\quad q=\frac{3}{4}=.75$

$np=40\cdot(.25)=10$ $\quad npq=40(.25)(.75)=7.5$ $\quad \sqrt{npq}=\sqrt{7.5}=2.739$

P(exactly 15 Correct ans.) = $P(x=15)$

$=$ binompdf(40, .25, 15)

$=.028$

P(at most 15 Correct Ans.) = $P(x\leq 15)$

$=$ binomcdf(40, .25, 15)

$=.974$

Nov 21-6:28 AM

Consider a binomial prob. dist. with $n=250$ and $p=.4$.

1) $q=1-p=.6$ $\quad$ 2) $np=250(.4)=100$ $\quad$ 3) $npq=250(.4)(.6)=60$

4) $\sqrt{npq}=\sqrt{60}\approx 7.746$

5) $P(x=120)=$ binompdf(250, .4, 120) $=.002$

$\Leftrightarrow 1-.994$

6) $P(x<120)=P(x\leq 119)=$ binomcdf(250, .4, 119)

Nov 21-6:38 AM

$P(x = a) = \text{binompdf}(n, p, a)$

$P(x \le a) = \text{binomcdf}(n, p, a)$

$P(x < a) = \text{binomcdf}(n, p, a-1)$

Consider a binomial Prob. dist. with $n=400$ and $p=.8$

1) $q = 1-p = .2$ 2) $np = 400(.8) = 320$ 3) $npqp = 400(.8)(.2) = 64$

4) $\sqrt{npqp} = \sqrt{64} = 8$

5) $P(x = 325) = \text{binompdf}(400, .8, 325) = .042$

6) $P(x \le 330) = \text{binomcdf}(400, .8, 330) = .907$

7) $P(x \ge 310) = 1 - P(x \le 309) = 1 - \text{binomcdf}(400, .8, 309) = .904$

we don't want | we want

309 310

Nov 21-6:47 AM

$P(x = a) = \text{binompdf}(n, p, a)$

$P(x \le a) = \text{binomcdf}(n, p, a)$

$P(x < a) = \text{binomcdf}(n, p, a-1)$

$P(x \ge a) = 1 - \text{binomcdf}(n, p, a-1)$

Nov 21-6:57 AM

Suppose prob. of any passenger show up for flight is .85.

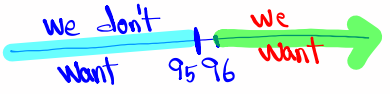
We randomly select 120 passengers.

1) $n = 120$ 2) $p = .85$ 3) $q = 1 - p = .15$

4) $np = 120(.85) = 102$ 5) $npq = 120(.85)(.15) = 15.3$ 6) $\sqrt{npq} = \sqrt{15.3} = 3.912$

7) $P(x < 110) = P(x \leq 109) =$ binomcdf(120, .85, 109) $= .978$

8) $P(x > 95) = P(x \geq 96) = 1 - P(x \leq 95)$



$= 1 -$ binomcdf(120, .85, 95)

$= .947$

Nov 21-7:12 AM

$P(a \leq x \leq b) =$ binomcdf(n, p, b) − binomcdf$(n, p, a-1)$

Keep b; Reduce by 1

Using last example

$P(95 \leq x \leq 110) =$ binomcdf$(120, .85, 110)$ − binomcdf$(120, .85, 94)$

Keep 110; Reduce by 1

$= .958$

Nov 21-7:22 AM

You are guessing on a True/False exam with 100 questions. Let x be # of correct answers. Success is # of correct Ans.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $np = 100(.5) = 50$ 5) $npq = 100(.5)(.5) = 25$ 6) $\sqrt{npq} = \sqrt{25} = 5$

7) P(exactly 60 correct answers)

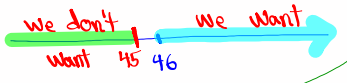
$P(x = 60) = \text{binompdf}(100, .5, 60) = \boxed{.011}$

8) P(fewer than 60 correct answers) $= .972$

$P(x < 60) = P(x \leq 59) = \text{binomcdf}(100, .5, 59) =$

9) P(more than 45 correct answers)

$P(x > 45) = P(\text{at least } 46) = P(x \geq 46)$



$= 1 - P(x \leq 45)$

Total Prob.

$= 1 - \text{binomcdf}(100, .5, 45)$

$= \boxed{.816}$

Nov 21-7:29 AM

10) P(get between 40 and 60, inclusive, Correct Ans)

Reduce by 1

$= P(40 \leq x \leq 60)$

Keep

$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

$= \boxed{.965}$

Nov 21-7:40 AM

Binomial Prob. Dist:

Mean μ $\quad \mu = np$

Variance σ^2 $\quad \sigma^2 = npq$

Standard deviation σ $\quad \sigma = \sqrt{\sigma^2}$

Consider a binomial Prob. dist with $n = 80$ and $p = .75$

$q = 1 - p = .25$ $\quad \mu = np = 80(.75) = 60$ $\quad \sigma^2 = npq = 80(.75)(.25) = 15$

$\sigma = \sqrt{\sigma^2} = \sqrt{15} \approx 3.873$

Round μ & σ to a whole #, then find

$\mu = 60$ $\quad \sigma \approx 4$

68% Range $= \mu \pm \sigma = 60 \pm 4 =$ 56 to 64

95% Range

Usual Range $= \mu \pm 2\sigma = 60 \pm 2(4) =$ 52 to 68

Keep

$P(52 \le x \le 68) = \text{binomcdf}(80, .75, 60) - \text{binomcdf}(80, .75, 51) = .973$

Reduce by 1

Nov 21-7:43 AM

You are making random guesses on a multiple-choice exam with 60 questions, each question has 5 choices, with only one correct choice. Success is to guess correct answer.

1) $n = 60$ $\quad$ 2) $p = \frac{1}{5} = .2$ $\quad$ 3) $q = \frac{4}{5} = .8$

4) $\mu = np = 12$ $\quad$ 5) $\sigma^2 = npq = 9.6$ $\quad$ 6) $\sigma = \sqrt{\sigma^2} = 3.098$

Round μ & σ to a whole #, then find

$\mu = 12$ $\quad \sigma \approx 3$

7) 68% Range $= \mu \pm \sigma = 12 \pm 3 \Rightarrow$ 9 to 15

8) 95% Range $= \mu \pm 2\sigma = 12 \pm 2(3) \Rightarrow$ 6 to 18

9) P(get between 6 to 18 correct Ans, inclusive)

Keep

$P(6 \le x \le 18) = \text{binomcdf}(60, .2, 18) - \text{binomcdf}(60, .2, 5)$

Reduce by 1

$= .966$

SG 16 ✓✓

SG 17 ⟶ You need to watch two videos, and do SG 17.

Nov 21-7:55 AM

Consider a binomial Prob. dist. with $n=60$, $p=\frac{1}{6}$.

1) $q = 1-p = \boxed{\frac{5}{6}}$

2) $\mu = np = \boxed{10}$

3) $\sigma^2 = npqp$
$= 60 \cdot \frac{1}{6} \cdot \frac{5}{6}$
$= \boxed{\frac{25}{3}}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{25}{3}} \approx 2.887 \approx \boxed{3}$

5) 68% Range
$= \mu \pm \sigma$
$= 10 \pm 3 \Rightarrow \boxed{7 \text{ To } 13}$

6) 95% Range
Usual Range
$= \mu \pm 2\sigma$
$= 10 \pm 2(3) \Rightarrow \boxed{4 \text{ to } 16}$

7) $P(5 \le x \le 15) = \text{binomcdf}(60, 1/6, 15) - \text{binomcdf}(60, 1/6, 4)$
Keep
Reduce by 1
$= \boxed{.946}$

Nov 21-8:09 AM

8) P(at most 2/3 Successes)

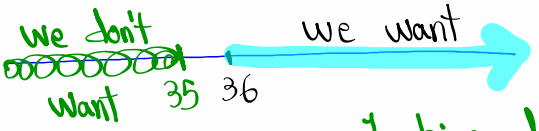
$\frac{2}{3}$ of $60 = \frac{2}{3} \cdot 60 = 40$

$P(x \le 40) = \text{binomcdf}(60, 1/6, 40) \approx 1$

9) P(at least 3/5 Successes)

$\frac{3}{5}$ of $60 = \frac{3}{5} \cdot 60 = 36$

$P(x \ge 36) = 1 - P(x \le 35)$



$\approx \boxed{0}$

$= 1 - \text{binomcdf}(60, 1/6, 35) =$

Nov 21-8:18 AM